For publication in the AIAA Journal

(NASA TM X 51242)

SOME EFFECTS OF PLAN-FORM MODIFICATION ON THE SKIN-FRICTION DRAG

N65-88999
X-64-10780

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[1957] 7p ref.

7p 60v1109

Code 2A

Submitted for Publication

For the rapid estimation of the skin-friction drag of airplane wing and control surfaces, it is sometimes the practice to use the Reynolds number based on the average chord of the surface in the calculations. This note will show that, if a simple average geometric chord is used, such an approximate solution can differ significantly from the more exact solution involving a spanwise integration of the skin friction.

The general plan form of a half-wing to be analyzed is shown in Fig. 1 with some of the symbols which will be used. For a two-dimensional wing the average skin-friction coefficient can be represented by

$$C_F = K(R/l)^n c^n \qquad (1)$$

where

n exponent of skin-friction equation

K constant of skin-friction equation (considered herein to include compressibility and heat-transfer effects)

R/l unit Reynolds number

When the average skin friction obtained by Eq. (1) at each spanwise station is integrated over the half-wing span, s, and the

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result is divided by the dynamic pressure and by the plan area of the half-wing, we obtain a general expression for the skin-friction coefficient

$$C_F = \frac{K(R/l)^n 2^{n+1}(c_{av})^n}{n+2} \left\{ \frac{\lambda_2}{(1+\lambda_2)[1-(b/s)] + (1+\lambda_1)(\lambda_2)(b/s)} \right\}^{n+1}$$

$$\left\{ \left[\frac{1-(\lambda_1)^{n+2}}{1-\lambda_1} \right] \frac{b}{s} + \left(1 - \frac{b}{s} \right) \left[\frac{1-(\lambda_2)^{n+2}}{(1-\lambda_2)(\lambda_2)^{n+1}} \right] \right\} \qquad (2)$$

for which the following limits apply

$$0 \leq \lambda_1 \leq 1 \; ; \qquad 0 < \lambda_2 \leq 1 \; ; \qquad 0 \leq b/s < 1$$

where by L'Hospital's rule

$$\lim_{\lambda_1 \to 1} \left[\frac{1-(\lambda_1)^{n+2}}{1-\lambda_1} \right] = \lim_{\lambda_2 \to 1} \left[\frac{1-(\lambda_2)^{n+2}}{(1-\lambda_2)(\lambda_2)^{n+1}} \right] = n+2$$

When there are no restraints on the factors contained in Eq. (2), the skin friction approaches 0 as $\lambda_2 \to 0$ - obviously an impractical solution. A family of plan forms is analyzed, therefore, which contains wings having the same ratio of average chord to root chord. To facilitate such an analysis, the notch ratio, b/s, in Eq. (2) will be written as

$$\frac{b}{s} = \frac{1 + \lambda_2 - (2c_{av}/c_{R_2})}{1 - \lambda_1\lambda_2} \qquad (3)$$

All wings will be assumed to be flying at the same unit Reynolds number and Mach number and to have the same average chord, span, and area. If we consider only wings with $\lambda_1 = 0$, then from Eqs. (2) and (3) we can write the equation of the skin-friction coefficient of the cranked wing to that for the untapered wing as

$$\frac{(C_F)_{\substack{\lambda_2 \\ \lambda_1=0}}}{(C_F)_{\lambda_1=\lambda_2=b/s=1}} = \frac{2^{n+1}}{n+2}\left(\frac{\lambda_2}{2c_{av}/c_{R_2}}\right)^{n+1}\left\{1 + \lambda_2 - \frac{2c_{av}}{c_{R_2}}\right.$$

$$\left. + \left(\frac{2c_{av}}{c_{R_2}} - \lambda_2\right)\left[\frac{1 - (\lambda_2)^{n+2}}{(1 - \lambda_2)(\lambda_2)^{n+2}}\right]\right\} \qquad (4)$$

The results of varying the inboard taper ratio, λ_2, in Eq. (4) on the skin friction of cranked wings to that for a rectangular wing are presented in Fig. 2 for an assumed $2c_{av}/c_{R_2} = 1/2$ for both the turbulent ($n = -1/7$ for large Reynolds numbers, see Ref. 1) and the laminar ($n = -1/2$) boundary-layer cases. Wing plan forms are also shown in Fig. 2 above each corresponding taper ratio. For the conditions specified, it is apparent that using the average chord in calculating the skin friction of such plan forms can lead to sizable errors.

Wings having straight leading edges (no crank) will be considered next. This case can easily be derived from Eq. (2) by letting $b/s = \lambda_2 = 1.0$. Provided the same conditions are imposed on the tapered and the untapered wings, namely, that each wing has the same average chord, span, area, unit Reynolds numbers, and Mach number, then the skin-friction ratio for the tapered wing to that for the rectangular wing becomes

$$\frac{(C_F)_{\lambda_1}}{(C_F)_{\lambda_1=1}} = \frac{2^{n+1}}{n+2}\left(\frac{1}{1+\lambda_1}\right)^{n+1}\left[\frac{1 - (\lambda_1)^{n+2}}{1 - \lambda_1}\right] \qquad (5)$$

for the limit

$$0 \leq \lambda_1 \leq 1$$

where again

$$\lim_{\lambda_1 \to 1}\left[\frac{1 - (\lambda_1)^{n+2}}{1 - \lambda_1}\right] = n + 2$$

Effects of taper ratio variations on the above skin-friction ratio are shown in Fig. 3. The results indicate that for wings having straight leading edges and small taper ratios the skin friction cannot be obtained accurately from a Reynolds number based on its average chord.

REFERENCE

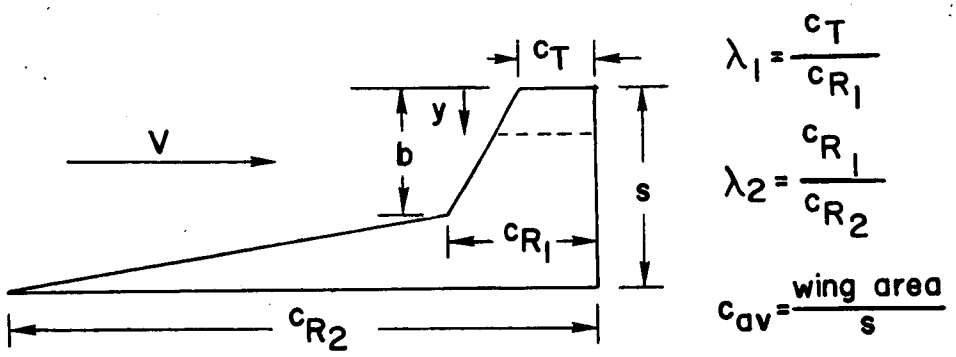
1. Locke, F. W. S., Jr.: Recommended Definition of Turbulent Friction in Incompressible Fluids. Bur. Aero., Navy Dept., (Design) Res. Div., DR Rep. 1415, 1952.

FIGURE LEGENDS

Fig. 1.- Geometry and symbols.

Fig. 2.- Ratios of the average skin-friction coefficient for cranked wings to rectangular wings.

Fig. 3.- Ratios of average skin-friction coefficient for tapered wings to rectangular wings.



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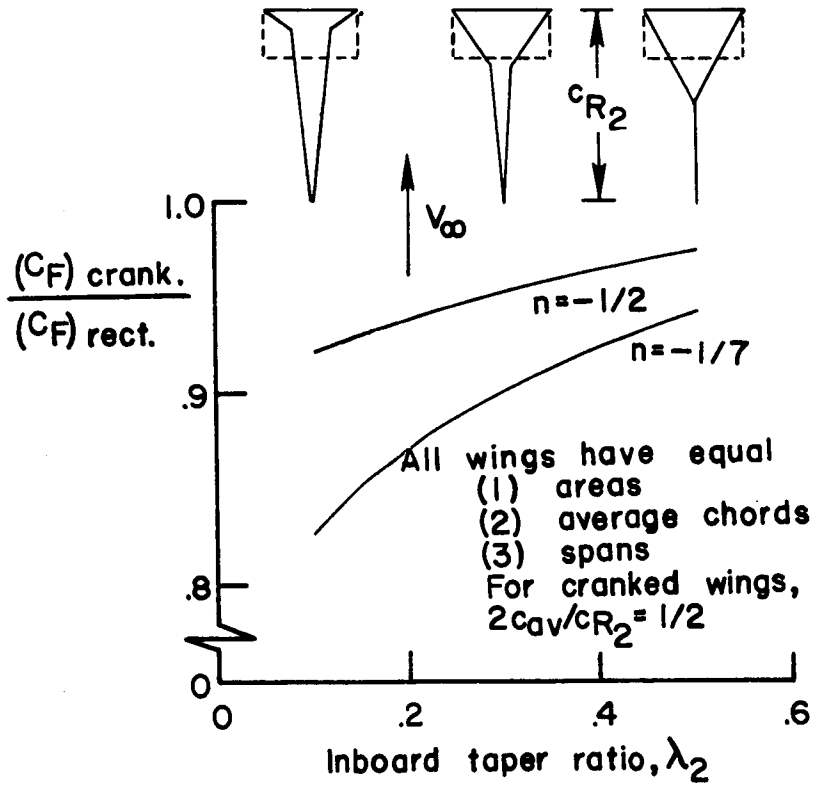

Fig. 2.- Ratios of skin-friction coefficient for cranked wings to rectangular wings.

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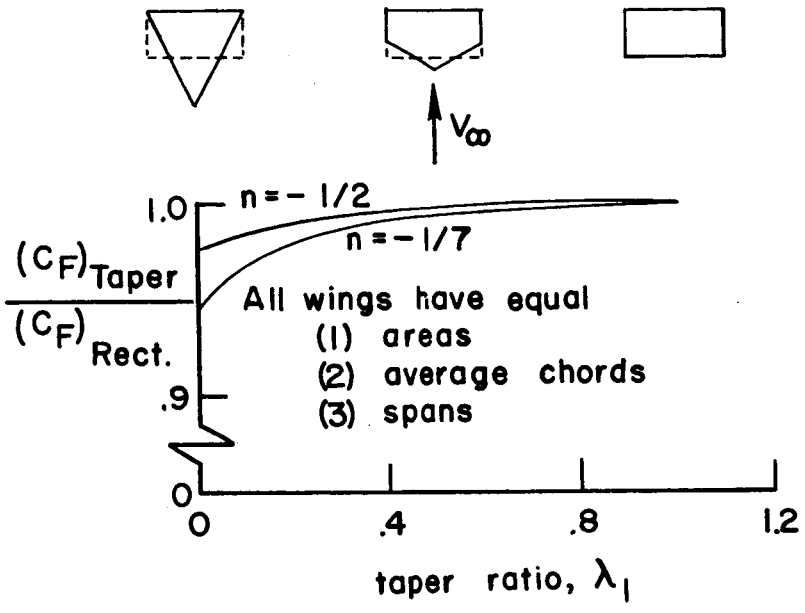

Fig. 3.- Ratios of skin-friction coefficient for tapered wings to rectangular wings.

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